Good choices in index number production

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Abstract: We interpret the index number production as an over-identified problem, where we have an overdose of possible choices. We present some good choices for the five critical issues in the problem.

Many choices in the index number problem are *over-identified*. There does not exist the only one "correct choice" but instead we have several possible choices. The term *over-identification* is mostly used for different possible estimators of some unique parameter: we have more than enough (or an overdose of) information for its estimation.

This may sound a rather trivial interpretation of the difficulties of decision in the case of index numbers, where we do not yet have generally accepted methods to evaluate the possible choices. In statistical estimation theory we have generally accepted criteria to evaluate the possible estimators and to choose the most reliable[1]. We believe that now we have good possibilities to approach this happy situation also in the case of index numbers.

We assume all the time that we have *complete information* provided e.g. by the weekly scanner data aggregated over consumers. Here all values, quantities and prices are based on (finest) homogenous commodities in chosen time periods. Indexes are binary comparisons, either for quantities or rather for prices. Binary comparisons for values are self-evident. We do not strive here for completeness/, but rather try to highlight the essential points and choices. There are five interrelated issues.

- Index number formulas
- Strategies for constructing index series
- Vanishing and new commodities, so-called null-commodities
- Time-aggregation
- Commodity-aggregation

[1] The population mean is normally estimated by the sample mean of n observations. Alternative choices are e.g. sample means of only n-1 observation (when one random observation is disregarded) or the *censored mean* of n-2 observations, where the minimum and maximum are disregarded. These both estimate the population mean well in large samples, but slightly less accurately than the ordinary mean.

Our views of these important issues and their preferred choices are given below. We just give the most evident competitors and leave the final choice for you.

1. Index number formulas

- Never use any *contingently biased* formula (like the basic indices or their FA´s or TA´s), if the data allows using *excellent* formulas. Simple excellent formulas include F, E, St, T, W, SV, MV, …
- The three sets excellent, permanently biased and contingently biased formulas are defined in Vartia & Suoperä (2018).
- Because prices vary much less than quantities, concentrate on price indices and avoid their FA’s (say FA of log-Laspeyres *l* = value ratio divided by *l(q)* or T(p) and not its FA).
- Use some index number *consistent in aggregation CA*, like MV. This allows natural and easy treatment of any sub-groups of indices.
- Use a formula which takes into account the null-values automatically or which *adapts to (or merges) null-values*, such as MV or SV. They do not react at all if null-values are imputed explicitly with small values and arbitrary prices (and the small values are made to approach zeros).

2. Strategies for constructing index series

- It is important to decide, what periods are compare directly using an index number formula and for what periods the changes are calculated indirectly using these previously calculated direct figures. The former directly calculated index numbers define the *strategy for constructing the index* series. Next, we define some new concepts, which allows a precise treatment of these important matters.
- For instance, yearly changes calculated from the yearly base index are not direct price-links, but derived changes. In the base strategy (or method) the price-links are based on comparisons $0 \rightarrow t, t = 1, 2, \ldots$, while in the chain strategy the direct price-links are based on $t-1 \rightarrow t, t = 1, 2, \ldots$. These two are the simplest strategies to construct index series and the actual strategies are normally some mixtures or combinations of them. A strategy hard to beat for monthly index could be based on links like $2016 \rightarrow 2017.m, m = 1, 2, \ldots, 12$ and $2016 \rightarrow 2017$ etc. Here all the months m of 2017 are compared to the monthly average of the previous year 2016 (i.e. using base strategy and Montgomery-Vartia) and the comparison period is changed by linking the consecutive years $2016 \rightarrow 2017$ (i.e. a chain strategy and Montgomery-Vartia). Here the base links like $2016 \rightarrow 2017.m$ do not suffer either contingent biases nor circular errors and they also contain maximal sets of commodities. Notice, how price-links are basic elements on which the strategies of constructing price index series depend on. Note also, that any strategy can be chosen totally independently of the choice of index number formula f. The index number formula f and the strategy (choice of links) are the two fundamental decisions in the index number construction. This almost self-evident distinction has been poorly treated and it is mostly lacking (except the pure base and chain strategies) in index number literature, see e.g. Vartia (1976, 2010).
- Consider some price index formula f comparing period 0 to the period 1 or $P_0^1 = f\begin{pmatrix} p^1 & q^1 \\ p^0 & q^0 \end{pmatrix}$. We say that a *price-link* from a period 0 to the period 1 is defined. The price-link is a *direct binary comparison* based on an index number formula from 0 to 1, in this direction. The point in the link is in the *periods* for which the calculation is made. These vary, while the index number formula f stays normally the same.
- To stress our point, the *links* say for which periods the selected price index formula f is applied. All other calculations are derived ones. Choice of links determines the *strategy* of construction in the index series.
- We illustrate the possible (or rather probable) incompatibility of the three-chain here. Suppose that commodity value shares in two consecutive months (say November and December) differ

considerably and systematically. This is due to the Christmas effect and cannot be explained by price or income changes. Therefore, static demand theory for these months cannot hold and is rejected. The same static demand functions with random errors do not explain the data for, say year 2016, November, 2017 and December, 2017 and these three periods suffer from *contingent chain error.* Simply said, the *derived* monthly price change between November, 2017 and December, 2017 calculated from the base index and from the *actual* binary comparison between the months differ from each other. These three binary comparisons for these periods are incompatible because the same static demand function does not explain their differences.

- Calculate base indices from some period of years length for months. These do not suffer from *contingent chain error* as chain indices do. Use some excellent formula.
- Calculate chain indices or some mixture of chain and base indices (mixed strategy) for years. Simple choices include chains from previous calendar year or chains over two years. Use some excellent formula.
- On the annual basis the static consumer theory with random errors cannot be easily rejected and contingent chain drift is probably small. This rather abstract motivation for yearly chain indices is the simplest explanation I have found this far.
- This rather abstract condition is based on the theory of Divisia-Törnqvist line-integral formula. First this idea based on continuous and differentiable prices and especially quantities is totally unrealistic (contrary to similar ideas in physics), unless price, quantity and values curves are smoothed. We propose considering moving weekly or monthly series for prices, quantities and values starting from every midnight (and interpolated within days). This necessity has been neglected in literature, except in Vartia (1976, pp. 108-122). These line-integrals are path-independent only if prices and quantities are connected by some *homothetic* demand theory, see Vartia (1976, p.120-122). The same holds without homotheticity if the price-quantity vectors move on the same indifference surface, see Vartia (1983). Therefore, if the quantity index stays roughly constant, the condition for the discrete approximations (say monthly chain price index) of the DT-index being path-independent is, that *some static demand system* (with additive random errors for log-quantities) holds for all relevant periods. Because of non-homogeneity, different discrete approximations and rather large random errors, the *contingent identity error* (or chain error) must be only approximately zero if the same static preference order is able to describe the (p,q,v)-vectors for all considered (say three) periods.
- Despite the highly abstract nature of the above criterium of the chain index being free of the *contingent identity error* or *contingent chain error*, we shall demonstrate, that it works astonishingly well for actual index number experiments. We have found no simpler criteria than this to make sense, what happens in the chain index (or more generally mixed strategy) calculations.
- Consider three periods called 0, 1 and 2 (not necessary in time order) and calculate their price changes P_0^1, P_1^2 and P_2^0 using the same formula satisfying TRT. These binary comparisons are mutually compatible or chain together if $P_0^1 P_1^2 P_2^0 = 1$. This holds not exactly but only accurately, if some static consumer demand system (with random errors) holds for all three periods. Approximation arises because of random errors in log-quantities, discrete approximations of DT-index and (least of all) non-homothetity. If we find significant price changes not explained by any static consumer demand system (with random errors), then $P_0^1 P_1^2 P_2^0 \neq 1$ or $P_0^1 P_1^2 \neq P_0^2$ or the three changes are *incompatible.* No static preference order holds for the three periods. In this way, price index calculations based on complete micro data reveal previously unseen limitations of demand theory. This discovery may have wide implications in economic theory.

3. New and vanishing commodities, so-called null-values

- Use some excellent index number which adapts to null-values, like SV or MV. They take automatically into account the new and vanishing commodities. For instance, MV does not react at all as a price index if null values are imputed with small values and arbitrary prices (and the values approach zero).
- This follows from the fact that null-values clearly have natural null-weights in MV. All the effects of the null values are directed to the quantity index and these are exceptionally large positive (respectively negative) values for new (vanishing) commodities.
- An index number should adapt to null-values. If values are null for both periods, they have no effect to any index numbers. If only one of the values is null, it should likewise have no effect. Of course, null-values should be minimized (e.g. when choosing the construction strategy) to some degree, because they carry only value and quantity information.
- None of the basic indices adapts to null-values. Most excellent indices such as ST, F or E do not adapt to null-values, while e.g. SV and MV do.

4. Time-aggregation

- Use years whenever feasible and synthetic months (best four or five calendar weeks) otherwise. Their values and quantities on the finest COICOP-level are calculated as simple sums and prices are their unit values.
- Calculate yearly indices by chain principle or use mixed strategy comparing t-2 and t. Use some excellent formula, such as SV or MV.
- In *the monthly base index*, change the base year every July from t-2 to t-1 and calculate base indexes from it to the synthetic months. Do not use the chain index for he months, though it is a good choice for years.
- Do not chain months with each other, because seasonal variations within months would invalidate the static demand theory and chain drift would appear.
- The problem in a chain is *not* in its binary comparisons, but in the multiplications (or divisions) of them. These are not consistent with the corresponding binary comparisons. This takes some time to appreciate

5. Commodity-aggregation

- In order to avoid quality changes, do not aggregate heterogenous commodities by summing quantities and calculating unit values for them (for higher levels of COICOP). Calculation unit values is a valid operation in time-aggregation (in time over commodities on the finest COICOP-level), but not over commodities.
- Using consistently aggregating CA indices (esp. ST and MV) allows any subgroups as intermediate stages of aggregation and their aggregation by the same formula leads to the correct overall index.
- Use index numbers, like L (as a price or quantity index) or rather ST or MV instead of unit values to aggregate commodities.
- This is the type of aggregation for which the index numbers were developed!

References:

Vartia, Yrjö (1976): **Relative changes and index numbers,** Research Institute of the Finnish Economy ETLA, series A4 (dissertation in statistics).

Vartia, Yrjö (1976b): Ideal log-change index numbers, *Scandinavian Journal of Statistics*, 3, 121-126.

Vartia, Yrjö (1978): Fisher´s five-tined fork and other quantum theories of index numbers, *In Theory and applications of economic indices* (Ed. W. Eichhorn, R. Henn, O. Opitz, R. W. Shephard), Physica-Verlag, Wurzburg.

Vartia, Yrjö (1983): Efficient methods of measuring welfare change and compensated income in terms of ordinary demand functions, *Econometrica*, 51,79-98.

Vartia, Yrjö (2010): Principles of defining index numbers and constructing index series, *HECER Discussion Papers No. 283, January 2010.*

Vartia, Yrjö & Suoperä, Antti (2018): Contingently biased, permanently biased and excellent index numbers for complete micro data.

Vartia, Yrjö and Vartia, Pentti (1984): Descriptive index number theory and the Bank of Finland currency index*, Scandinavian Journal of Economics,* 86, 3, 352-364.