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Introduction

This methodological description presents the method used by Statistics Finland as a basis for its productivity calculations. The purpose is to serve users who would like to know more about our calculation method. Although we have made every effort to keep the presentation simple and easy to approach, the complexity of the matter poses challenges in this respect.

You can read more about the method used in productivity surveys in the OECD manual Measuring Productivity. Statistics Finland follows its recommendations in its productivity survey, but there are minor differences between the methods. Furthermore, the manual takes a rather theoretical approach to the matter. We have therefore determined that the method used by Statistics Finland should be described.

Measuring productivity is essential when factors behind economic growth are studied. Statistics Finland has published statistics on productivity surveys since 2005, and the time series extends back to 1975. Growth accounting based on the KLEMS method is used to produce productivity statistics, and most of the calculations are based on the national accounts data. Statistics Finland also uses the employment statistics and the Labour Force Survey to examine the quality of labour input.

The KLEMS method used in the productivity surveys is recommended by the OECD in its productivity manual and the method has been developed by such scholars as Dale Jorgenson, Professor at Harvard University. KLEMS stands for capital (K), labour (L), energy (E), materials (M) and services (S). These are production inputs, and the method can be used to examine their impacts on productivity. However, in productivity research, only the impacts of capital and labour are examined.

1 What is meant by productivity?

Productivity is the output-to-input ratio in the production process. In other words, it describes the output generated with a specific input.

$$Productivity = \frac{output}{input} \qquad (1)$$

The equation shows that productivity grows when output increases or when input shrinks. GDP per hours worked is the most commonly used productivity indicator in the national accounts. In this indicator, GDP is the output, and the hours worked is the input.

$$Labour\ productivity = \frac{value\ added}{hours\ worked} \qquad (2)$$

The value added and hours worked can be directly obtained from the national accounts. In other words, when labour productivity increases, fewer inputs (hours worked) are needed to generate more end results (value added). This means that productivity is closely connected to both standard of living and competitiveness.

The link with the standard of living arises from higher labour productivity. When it increases, more output can be generated during the same number of hours worked. The output is available to people as goods and services. Therefore, even if we work shorter hours, we can still produce the same number of goods and services, and the standard of living remains unchanged. At the same time, competitiveness also improves as the same number of inputs generate more output than our competitors.

In other words, the aim is to boost productivity and to examine how it changes year on year and in the longer run, or which sectors are better at improving productivity than others.

The aim is to break labour productivity into its components so that we can determine the reasons for productivity growth. We can start by examining the growth in value added and the factors contributing to its growth. Value added arises from labour input, capital input and technological improvements. In productivity calculations, all three factors are combined into multi-factor productivity.

$$\begin{aligned} Change\ in\ value\ added &= (share\ of\ labour\ input) * change\ in\ labour\ input \\ &+ (share\ of\ capital\ input) * change\ in\ capital\ input \\ &+ change\ in\ multi-factor\ productivity \end{aligned} \qquad (3)$$

It should be noted that in these calculations, the change in labour input includes both change in hours worked and change in the quality of labour. The number of inputs cannot be increased indefinitely, which means that multi-factor productivity remains the key variable affecting growth in value added.

We can calculate the change in labour productivity by dividing the change in value added by the change in labour input:

$$\frac{Value\ added}{labour\ input} = (\text{share of labour input}) * \text{change in the quality of labour} + \text{share of capital input} * \left(\frac{capital\ input}{labour\ input}\right) + change\ in\ multi - factor\ productivity \qquad (4)$$

The residual left after the impacts of input growth have been removed from productivity growth cannot be explained with production inputs alone. In the productivity research method, this residual is known as multi-factor productivity. It can be attributed to technological improvements such as better production technologies, ideas and better management methods. One aim of productivity research is to determine the factors behind changes in multi-factor productivity.

The same matter is discussed in Section 2 with the help of equations.

Productivity surveys also examine trends in labour productivity from another perspective: instead of value added, the output of the national accounts is used as the output. In other words, intermediate consumption is also included. In the examination, change in output is also broken down into its components so that the impact of intermediate consumption on changes in output and labour productivity can also be examined.

2 Calculation method used in productivity surveys

2. 1 Change in labour productivity, as broken down into its components

In macroeconomic theory, labour force and capital are used as inputs to generate outputs. In this description, labour force (labour input) is measured by the number of hours worked. At the same time, it is also affected by the human capital of the employees (quality of labour input). This can be illustrated by the production function.

$$Y = AK^{v_K}(hL)^{v_L} \qquad (5)$$

Y stands for the amount of output, K for capital input, h for employees' human capital, and L for labour input. v_K and v_L describe the contributions of capital and labour inputs to production, and their sum must therefore be one. In practice, labour input is the contribution of labour compensations to value added. Only capital compensations are left after labour compensations have been subtracted from value added. The share of capital compensations of value added is the share of capital input.

In the equation, A stands for multi-factor productivity. Thus, in addition to traditional physical inputs and employees' human capital, other factors affect the output.

When we calculate labour productivity (Y/L), we get the following equation:

$$\frac{Y}{L} = A(\frac{K}{L})^{v_K} H^{v_L} \qquad (6)$$

We can now take both logarithms from the equation, because they make it easier to process the equation. Moreover, logarithmic rates of change can be added, which also makes the calculation easier. We can use Δ to mark the changes. The calculation produces the following results:

$$\Delta\, Ln\frac{Y}{L} = \Delta\, ln\, A + \mathrm{v}_k\, \Delta \ln\frac{K}{L} + v_L\, \Delta\, ln\, h \qquad (7)$$

In productivity surveys, the weights v_K and v_L are calculated as averages of two successive years. We added lines were added above the variables in question to show this. Other parts of the equation describe the changes.

We added sub-indices j, referring to a specific industry, to the equations.

$$\Delta\, ln\, \frac{Y_j}{L_J} = \Delta\, ln\, A_j + \overline{v}_{K,j}\, \Delta \ln\frac{K}{L} + \overline{v}_{L,j}\, \Delta\, ln\, h_j \qquad (8)$$

This equation is used in productivity surveys to examine labour productivity.

Only A remains to be determined after we have obtained Y (amount of output or value added), L (hours worked) and K (capital) from the national accounts, and h (human capital) from other sources.[1]

Thus, in productivity surveys, all terms of the equation, except for A (multi-factor productivity) can be obtained from statistical sources. A is therefore determined using equation 8, and it is calculated as a residual. Thus, change in multi-factor productivity can be determined by subtracting changes in inputs from changes in labour productivity.

It should also be noted that in equation 8, K (capital) is shown as divided by L (labour input). Term K/L describes capital intensity, which is the ratio of capital to labour input (hours worked). Thus, we must calculate the contribution of capital intensity to labour productivity.

h_j or employees' human capital is the only factor remaining in the last term of labour input.

The results of the calculations can be viewed on the website of the statistics on productivity surveys (in the database table showing components of productivity change (value added)).

[1] Employment statistics and Labour Force Survey

2.2 Change in value added, as broken down into its components

By changing the production function in equation 1, we can also examine the breakdown of change in value added into its components. However, in this case, we do not need to divide the equation by the amount of labour input and thus

$$\Delta\, Ln\, Y_j = \Delta\, Ln\, A_j + \overline{v}_{K,j}\, \Delta \ln K_j\ + \overline{v}_{L,j}\ \Delta\, ln\, L_j \qquad (9)$$

The following sections describe in more detail how the labour input and capital input affecting changes in productivity and value added are calculated.

The results of the calculations can be viewed on the website of the statistics on productivity surveys (in the database table showing components of change in value added).

2.3 Components of change in labour productivity calculated from output

Labour productivity can also be calculated based on production. In that case we calculate the change in production per hours worked, and break it down to components. The equation then becomes

$$\Delta\, ln\, \frac{Y_j}{L_J} = \ \Delta\, ln\, A_j\ +\ \overline{v}_{K,j}\, \Delta \ln \frac{K}{L}\ +\ \overline{v}_{M,j}\, \Delta \ln \frac{M}{L}\ +\ \overline{v}_{L,j}\ \Delta\, ln\, h_j \qquad (10)$$

M_j is intermediate consumption. Its weight is the share of intermediate consumption in production.

The results of the calculations can be viewed on the website of the statistics on productivity surveys (in the database table showing components of productivity change (output)).

2.4 Change in output, broken down into its components

The change in production can also be broken down to different components and in that case also the component of intermediate consumption can be taken into account.

$$\Delta\, ln\, Y_j = \Delta\, ln\, A_j + \overline{v}_{K,j}\, \Delta \ln K_j + \overline{v}_{M,j}\, \Delta \ln M_j\ + \overline{v}_{L,j}\ \Delta\, ln\, L_j \qquad (11)$$

M_j is intermediate consumption. Its weight is the share of intermediate consumption in production.

The results of the calculations can be viewed on the website of the statistics on productivity surveys (in the database table showing components of changes in output).

3 Labour input and capital input

It was concluded above that the amount of labour input and capital input can be obtained from the national accounts. With regard to labour input, this is true in the sense that in productivity surveys, the amount of labour input is measured on the basis of the hours worked. However, we also need other information to measure the quality of labour input (human capital). Measuring labour input and its quality is described in more detail in Section 4.

With regard to capital input, the national accounts describe the amount of capital in the economy. Information on prices is also available, because we know the prices of investments. However, to examine productivity and calculate the contribution of capital input, we must obtain more detailed national accounts data and produce additional calculations on their basis. Calculation of capital input is described in more detail in Section 3.5.

Table 1 describes the content of labour input and capital input, and makes comparisons between them. Labour input and capital input are two different quantities, and they are calculated differently. However, they should be made commensurable in productivity calculations so that the contribution of both factors to production can be determined. As stated above, labour input is measured on the basis of the hours worked. However, the amount of capital input is measured on the basis of capital services. This means that the contribution of capital input to production can be better described by the transport service produced by a motor vehicle than the number of the vehicles themselves, for example. (For a more detailed description of the calculations, see Section 4).

Labour compensation per hours worked constitutes the price of labour input (labour compensation is a slightly broader concept than wages and salaries). With regard to capital, we base our calculations on the user cost of capital, using it as the price of capital input.

Both input types are classified under two-digit level industries, but labour input is also classified under quality categories (age, educational level), while capital input is also classified under assets (buildings, machinery and equipment, etc.).

Labour cost (which is also labour income) can be obtained by multiplying the number of hours worked by the labour compensation paid for the hours worked. Correspondingly, the total cost of capital input can be obtained by multiplying the number of capital services by the user cost.

We also need aggregation weights so that we can determine the changes in the amount of capital input or labour input by industry. When labour inputs of different quality are weighted together, industry-specific and quality category-specific shares of total compensations are used as aggregation weights. Correspondingly, we can calculate the change in capital input by weighting different assets with industry-specific and asset-specific shares of user costs of the whole economy and the industry as a whole.

Table 1. Labour input and capital input.

	Labour input	**Capital input**
Basic quantities	Human capital	Fixed capital
Services provided by inputs for production:		
Amount	Hours worked	Capital services

Prices	Labour compensation per hours worked	User cost of capital per capital service unit
Classification	Industry, types of job categories	Industry, asset
Input price/input income	Labour compensation per hours worked* total hours	User cost * capital services
Aggregation weights	Industry-specific and quality category-specific shares of the sum of labour compensation	Industry-specific and asset-specific shares of user costs

Source: Measuring Productivity. OECD Manual. OECD 2001.

4 Calculating labour input and its components

Labour input is one of the terms used in growth accounting, and it can be further broken down into two different components: the change in the quality of labour input[2] and the quality-standardised change in the number of hours worked. Labour input plays an important role, because as the impacts of labour input and capital input are subtracted from value added, multi-factor productivity is left as a residual. Examination of labour input as such is also interesting, because we can distinguish between total labour input and the change in the structure of labour input. The latter describes such trends as changes in the skills of the labour force.

In productivity surveys, labour input is measured on the basis of the hours worked obtained from the national accounts.

However, in the calculation of labour input, it is also important to note the change in the quality of labour input. This is because the labour input that is only based on the changes in the hours worked does not reflect the differences in the productivity of labour input categories. If labour input is only measured on the basis of the changes in the hours worked, all employees would be treated on an equal basis. This means that the hours worked by such persons as an experienced surgeon and a teenager working in a fast-food restaurant as a part-timer are considered of equal value. Basically, it fails to take into account the educational level, experience and position of the employee. However, the wage and salary details obtained indicate that the value of an hour worked by a highly educated and experienced employee differs from the value of an hour worked by a younger and less educated employee (Jorgenson et al, 2005). This adds to the economic heterogeneity of the examination of hours worked. The hours worked should therefore be grouped on the basis of the qualities of individual employee categories so that we can create a labour force quality indicator taking into account differences between employees (Jorgenson et al, 2005). In its

[2] Change in the quality of labour input can also be called structural change in labour input. It can be considered change in employees' human/intellectual capital.

productivity surveys, Statistics Finland uses ‘change in the quality of labour input’ as the variable describing the heterogeneity of the labour force. The manner in which changes in the quality of labour input are measured is described in more detail below.

Quality-adjusted labour input (QALI) used in productivity surveys consists of two parts: the sum of changes in unweighted hours worked and in the quality of labour input. The change in unweighted hours worked can be easily calculated by industry or at the level of the entire economy by comparing the number of hours worked during the current year with the number of hours worked in the previous year. However, the quality of labour input cannot be directly observed, and it is therefore calculated as a residual by subtracting the change in unweighted hours worked from the change in weighted hours worked.

Quality-adjusted labour input is obtained on the basis of the hours worked by employees and the labour compensations paid for their work. To calculate the change in the quality of labour input, we must first divide the hours worked into structural groups. Structural groups refer to the fact that the hours worked are divided into different groups on the basis of their qualities. Labour force qualities are defined on the basis of such features as the person’s educational level, gender and age. A total of nine different structural groups are used in the calculation of labour input in productivity surveys.

*Table 2. Labour composition groups (3*3=9)*

	Number of classes	**Divisions**
Age	3	1. < 30 years, 15–29 years 2. 30≤ years ≤ 49, 30–49 years 3. > 49 years, 50+ years
Education	3	**1. HS, High skilled** Polytechnic degree Lower university degree Higher university degree Licentiate and doctorate degrees **2. MS, medium skilled** Matriculation examination Other secondary level education **3. LS, low skilled** Lower secondary education or unknown

Employment statistics and the Labour Force Survey are used as a basis for the composition groups. Employment statistics are annual statistics providing data by region on the population’s economic activity and employment. Employment statistics are released once a year and are completed always in October. Final statistics are completed with a delay of nearly two years. Thus, the final data for 2019 will be published in October 2021. The Labour Force Survey data collection is based on a random sample drawn twice a year from Statistics Finland’s population database. The survey is used as a basis for official statistics on such areas as employment and unemployment.

Both statistics also contain the details of individuals' socio-economic status (employment, educational level, age, etc.). The composition groups referred to in Table 1 are compiled, and the distributions of the composition groups at desired levels are calculated using both datasets as a basis.

The total labour input is thus created using the labour compensation and hourly compensation data contained in the national accounts, which are divided into composition groups on the basis of the distributions of the employment statistics and the Labour Force Survey. In accordance with the neoclassical theory, in the markets characterised by absolute competition, employees receive a compensation equivalent to their marginal productivity. Thus, productivity surveys are based on the assumption that the compensation paid to the employees reflects marginal productivity. The example below illustrates how the data contained in the national accounts are divided into composition groups based on the distributions of other sources.

Table 3. Distribution of hours worked and labour compensations in the national accounts into composition groups

A	B	C	D	E	F	G	H	I
Year	Industry	Composition group	Distribution: hours	Distribution: compens.	NA hours	NA compens.	Hours per group	Comepns. per group
Year2	j	1 HS	4 %	4 %			9	189
Year2	j	1 LS	4 %	2 %			9	111
Year2	j	1 MS	26 %	18 %			57	929
Year2	j	2 HS	15 %	20 %			33	1 031
Year2	j	2 LS	4 %	4 %			10	229
Year2	j	2 MS	23 %	24 %			51	1 265
Year2	j	3 HS	8 %	13 %			17	670
Year2	j	3 LS	4 %	4 %			10	217
Year2	j	3 MS	11 %	11 %			25	598
		Total	**100 %**	**100 %**	**221**	**5 238**	**221**	**5 238**

Columns H and I of Table 3 show the end result after the data contained in the national accounts for year t (year2) have been divided into labour composition groups.

The next stage in the creation of a quality-adjusted labour input involves the calculation of changes in hours worked for each composition group and the calculation of weights for each group using labour compensations. The annual change in the hours worked is calculated for each composition group as a logarithmic change (see Table 4a). The compensation weight is created by determining the two-year average share of each group's labour compensations of the overall level of compensations (see Table 4b).

Table 4a. Calculating changes in hours worked by each composition group.

A	B	C		D
		Hours per group		
Industry	Composition group	Year1	Year2	change, %
j	1 HS	10	9	-2.42 %
j	1 LS	9	9	-0.72 %
j	1 MS	58	57	-0.92 %
j	2 HS	35	33	-4.25 %
j	2 LS	10	10	-6.42 %
j	2 MS	52	51	-1.82 %
j	3 HS	17	17	2.10 %
j	3 LS	10	10	-9.28 %
j	3 MS	25	25	0.10 %
		226	**221**	**-1.97 %**

Table 4b. Weight based on labour compensations.

A	B	C		D		E	
		Comepns. per group		Shares of compens.		Compens. share weight	
Industry	Composition group	Year1	Year2	Year1	Year2		Year2
j	1 HS	191	189	0.04	0.04		0.04
j	1 LS	117	111	0.02	0.02		0.02
j	1 MS	925	929	0.17	0.18		0.18
j	2 HS	1 058	1 031	0.20	0.20		0.20
j	2 LS	243	229	0.05	0.04		0.04
j	2 MS	1 278	1 265	0.24	0.24		0.24
j	3 HS	669	670	0.13	0.13		0.13
j	3 LS	236	217	0.04	0.04		0.04
j	3 MS	589	598	0.11	0.11		0.11
		5 306	**5 238**	**1.00**	**1.00**		**1.00**

Changes in hours worked in each composition group are shown in Panel a of Table 4. The annual change in hours worked in industry *j* is shown on the last line. Note that the annual changes in composition groups are not summed up in changes in overall level (Column D of Table 4a). The shares of the labour compensations of each composition group of the total labour compensations of the year in question are shown in Panel b. The weight shown in Column E is the average of the shares of two successive years.

In Table 5, composition group-specific changes in hours worked (Column D of Table 5[3]) are weighted with the average weight of the compensations (Column E of Table 5). Following this, the data are summed up across composition groups. This is how the annual change in **quality-adjusted labour input** is obtained.

As stated above, quality-adjusted labour input consists of changes in unweighted hours worked and the change of labour quality component. The example above shows the stages of constructing a quality-adjusted labour input (the end result is shown in Column F of Table 5). In year 2, the change in the quality-adjusted (**weighted**) labour input was –1.95 per cent. The impact of each composition group on changes in labour input is also shown in Table 5.

From year 1 to year 2, the change in **unweighted** hours worked was –1.97 per cent (ln(221/226)). This is shown in Panel a of Table 4. The change in unweighted hours worked is shown as annual change in hours worked, and no consideration has been given to composition groups with different types of productivity. In this method, each hour worked is given equal weight, irrespective of the qualities of the employee. At the same time, the total labour input shown in Table 5 takes into account employee groups with differing qualities.

The change in the quality of labour input (composition change) cannot be directly measured, as it is calculated as a residual (difference between weighted and unweighted hours worked). In this case, the composition change in labour input in the sample year was 0.02 percentage points ([–1.95] – [–1.97]). Generally speaking, labour quality term is positive when the total labour input (weighted hours worked) grows more rapidly than unweighted hours worked. In the sample years, the annual change in unweighted hours worked (–1.97) was more negative than that of weighted hours worked (–1.95). Thus, the quality-adjusted labour input has declined less than indicated by the value measured directly on the basis of the year-to-year change. Between these years, there were (positive) changes in the quality of labour input that slowed down the decline in the number of hours worked.

[3] Figures in Table 5 are from Column D of Table 4a and Column E of Table 4b.

Table 5. Generation of quality-adjusted labour input.

A	B	D	E	F
		Change in hours	Compens. share weight	Contribution
Industry	Composition group	Year2	Year2	Year2
j	1 HS	-2.42 %	0.04	-0.09 %
j	1 LS	-0.72 %	0.02	-0.02 %
j	1 MS	-0.92 %	0.18	-0.16 %
j	2 HS	-4.25 %	0.20	-0.84 %
j	2 LS	-6.42 %	0.04	-0.29 %
j	2 MS	-1.82 %	0.24	-0.44 %
j	3 HS	2.10 %	0.13	0.27 %
j	3 LS	-9.28 %	0.04	-0.40 %
j	3 MS	0.10 %	0.11	0.01 %
			QALI	**-1.95 %**

As formulas, quality-adjusted labour input is calculated as follows:

$$\Delta lnL_{j,t} = \sum_{I} \bar{v}_{I,j,t} \, \Delta lnH_{I,j,t}$$

Formula (1) $\Delta lnH_{I,j,t}$ refers to composition group-specific changes in the number of hours worked (as shown in Column D, Panel a of Table 4). $\bar{v}_{I,j,t}$ is the average weight of two successive years based on the shares of labour compensations. Shares of total labour compensations have been calculated for each composition group (see Panel b of Table 4). Summing up across composition groups is shown in Table 5. Formula sub-index *j* refers to the desired calculation level. An industry or the entire economy can be used as the calculation level.

Formula to calculate labour quality component:

$$\Delta lnLC_{j,t} = \Delta lnL_{j,t} - \Delta lnH_{j,t}$$

Formula (2) $\Delta lnLC_{j,t}$ shows the change in labour composition, which reflects the qualitative change in labour input. It has been calculated as a residual and as the difference between total labour input and unweighted hours worked. $\Delta lnH_{j,t}$ is the change in the hours worked at desired level, depending on how sub-index *j* is defined.

5 Calculating capital input and its components

5.1 Capital input and change in its amount

To examine the impact of the change in labour input on labour productivity or value added, we need to know the amount of labour input. Likewise, to examine the impact of capital input, we need to know the amount of capital input. How can we obtain figures for capital input that are comparable with those for labour input?

In the national accounts, the amount of capital is measured on the basis of gross and net stocks. The net stock shows the amount of fixed (non-monetary) capital in productive use in the economy. The net stock is the accumulation of past years' investments from which consumption has been subtracted. The production capacity of the assets weakens as they age, and in addition to consumption, this is also taken into account in the calculation of productive capital stock. This does not necessarily take place in the same proportion or as rapidly or slowly as the consumption affecting the net stock. The age-efficiency profile describes the loss of productive capacity of productive capital stock. Thus, the age-price profile and the age-efficiency profile are not necessarily identical. For example, the price of a new car or other vehicle may decline substantially during the first year of use (age-price profile). However, its transport capacity remains unchanged (age-efficiency profile).

Productive capital stock produces capital services for the production process. Capital services describe the impact of the accumulated capital stock on the amount of production. Productive capital stock is the amount of accumulated capital stock, whereas capital services are a flow describing the amount of the capital service produced by capital stock assets. For example, the amount of the productive capital stock of machinery and equipment describes the amount of machinery and equipment used in the economy. Machinery and equipment produce capital services when carrying out tasks that could not be carried out without them. Buildings and structures also produce services (provide protection). Transport vehicles produce transport services.

One problem with measuring capital input is that the capital used in the production process is often owned by the capital user. This means that when capital generates capital services for its owner, acquisition and sales of the capital in the market go unnoticed. It thus differs from labour input: employees own their own labour input and sell it to enterprises in the labour market, and in such situations, a specific price for it (salary) is determined. It is difficult to determine a similar price for capital.

In addition to the amount of capital services, we need information on the price of capital services. These are known as user costs of capital. Prices for capital services could be determined in a market if such a market existed. However, as no such market exists, user costs are determined on the basis of investment prices, depreciation rates and the internal rate of return.

Table 4. Asset classifications used in productivity surveys and the national accounts.

Productivity surveys	**National accounts**
Residential buildings	N111 Residential buildings
Other buildings and structures	N112 Other buildings and structures
Machinery and equipment	N1131 Transport equipment, N1139 Other machinery and equipment
ICT capital	N11321 Computers and peripheral equipment, N11322 Other communications technology equipment, N1173 Computer software and databases
R&D capital	N1171 Research and development
Other capital	N114 Weapons systems, N115 Cultivated biological resources, N1174 Entertainment, literary and art originals, N1179 Mineral exploration

Productive capital stock and the user cost of capital are calculated for each asset each year and classified by industry. Logarithmic rates of change are calculated for productive capital stock each year. We also use the Törnqvist index: the share of user cost of each asset of the user cost of the assets of the industry is calculated for each year, and the average of the shares of the current and previous year is always taken.

A numerical example is given in the table below.

	Year 1			Year 2					
Asset	productive stock	user cost	share of user cost	productive stock	user cost	share of user cost	average of user cost shares	log change of productive stock	**change in capital input**
R&D assets	50	1	0.33	55	1	0.25	0.29	1.61	**0.47**
IT assets	60	2	0.67	70	3	0.75	0.71	2.30	**1.63**
total user cost		3			4				

In other words: when we combine the amount of capital (productive capital stock) and user cost of capital, we can calculate capital services (volume index of capital input). Part of the second term of the right side of equation 6:

$$\Delta \ln K_j = \sum_k \overline{v}_{j,k} \Delta \ln K_{j,k}$$

Thus, the change in the amount of capital input in industry j $\Delta \ln K_j$ can be obtained when changes in asset-specific productive capital stock $\Delta \ln K_{j,k}$ are weighted with the Törnqvist indices of user costs.$\overline{v}_{j,k}$

$$\overline{v}_{j,K}\ \Delta \ln K_{j,K}$$

In practice, two calculations are made: logarithmic changes for productive capital stock are calculated, and the share of each asset of the sum of the user costs of all assets for the user cost of capital is calculated. These two are then multiplied by each other (and by 100). To obtain the contribution of each asset to the change in value added, the change must also be weighted with the share of the asset user cost of the value added. As we calculate the contribution to labour productivity, we must also weight changes in asset capital input with the share of costs of use of the value added. This is discussed in more detail in Section 6.

5.2 User cost of capital

Let us take another look at the calculation of user cost of capital. Unlike in labour input, no distinction is made in capital input between capital assets of different qualities. However, a distinction is made between capital assets of different types. Each industry has different types of capital assets, such as buildings, machinery and R&D stock.

Before examining the impact of capital services (different types of asset) on productivity, we should first make different asset categories commensurable. This is done with the aid of prices. As shown above, we should calculate user costs of capital and use them to weight productive capital stocks of different assets.

User cost of a capital asset can be calculated with the following equation:

User cost of asset=internal rate of return+depreciation rate–change in investment price

or

$$p^I_{k,j} = p^I_{k,t-1} i_t + \delta_k \; p^I_{k,t} \; - [p^I_{k,t} - p^I_{k,t-1}]$$

The depreciation rate δ_k is included in the equation because assets losing their production capacity particularly rapidly (such as computers) are given more weight in capital services than such items as buildings (which depreciate more slowly). Thus, the impact of rapidly depreciating assets on capital services is greater than their share of capital stock. In other words, they are more expensive.

Change in the price of investments $p^I_{k,t} - p^I_{k,t-1}$ describes the change in the price of an asset during the year.

At the same time, however, the internal rate of returni_t is not directly known, and it must be calculated using a specific formula. It can also be called the (internal) rate of return. The term describes the alternative cost that arises when the owner is using the asset for productive purposes instead of spending an equal sum of money differently.

Formula to calculate the internal rate of return:

$$i_t = \frac{p^K_t K_t \; + \; \sum_K [p^I_{k,t} - p^I_{k,t-1}] K_{k,t} - \sum_K p^I_{k,t} \delta_K K_{k,t}}{\sum_K p^I_{k,t-1} \, K_{k,t}}$$

In the calculation of the internal rate of return, the amount of capital compensations is the first term of the numerator.

In practice, user costs are used as a distributor to distribute capital compensations to different assets. Capital compensations are obtained by subtracting labour compensations from value added. It follows from this that they can only be calculated by industry (not for each asset separately). The distribution of the costs of use among the assets can only be determined if we calculate the costs of use for each asset.

6 Contributions of labour and capital inputs

The calculation of changes in labour and capital inputs has been examined above. We must still examine how these changes have affected the change in value added and labour productivity. In other words, we must calculate the contributions.

Value added is considered to be divided into labour and capital compensations: the value added generated during production is shared between the inputs creating the value added. As stated above, the labour force receives this labour compensation in the form of a salary and other benefits, while for capital assets, it is a user cost. When we calculate their shares of value added, the shares can be used as weights of input change rates, and contributions to the weights can thus be calculated.

Thus, for capital input, we calculate the shares of user costs of value added for all assets in all industries. Again, the Törnqvist index is used: the shares of successive years are calculated, and their average is determined. For an example, see Table 7.

Table 5. Calculating weight of capital input

	Year 1			**Year 2**			
Asset	user cost	industry value added	user cost/value added	user cost	industry value added	user cost/value added	user cost/value added-törnqvist
R&D assets	20	100	0.2	30	120	0.25	0.23
IT assets	30	100	0.3	40	120	0.33	0.32

The changes have now been calculated as logarithmic rates of change and weighted with costs of use/value added weights. We should still calculate the shares of individual industries of the combined value added of all industries. When changes are multiplied by these weights, we can obtain the contribution of each industry to the equivalent change in the entire economy.

7 The reallocation of labour input between industries

In addition to above mentioned contributions, in Productivity surveys also the so called labour reallocation has been separeted. The reallocation of labour means that the labour reallocates to different industries, to industries that have higher productivity, or sometimes to industries that have lower productivity.

This can be shown in equation

$$\Delta \ln \frac{Y}{L} = \sum w_j \Delta \ln \frac{Y_j}{L_j} + R, \qquad (17)$$

where the reallocation of labour

$$R = \sum w_j \Delta \ln H_j - \varDelta \ln H = \sum w_j \Delta \ln H_j - \sum s_j \varDelta \ln H_j = \sum (w_j\, s_j) \Delta \ln \; H_j$$

(18)

So we calculate the changes in productivity for each industy, and separately for the sum of all industries. The difference between these is the reallocation of labour.

The results of the calculations can be viewed on the website of the statistics on productivity surveys (in the database table showing components of productivity change (value added)).

8 Calculation-related challenges and matters for consideration

In connection with the calculation of the amount of capital, there is also information on investments that indicate changing amounts of capital in the economy. However, to calculate the amount of capital, we also need assumptions of the lifetime and consumption of capital. This is also based on the assumption that the entire capital stock thus calculated is in productive use.

In these calculations, capital stock only includes capital that is measured in the national accounts. Human capital is also taken into account through quality of labour calculations.

Productivity indicators should not necessarily be examined on the detailed level of industries or by reviewing changes over a period of two years. The calculations are of such a nature that they reveal more about long-term productivity trends. For a variety of production-related reasons, value added in individual industries may change year on year. Such changes may not necessarily be any indicator of productivity, even though any major changes in value added would inevitably also lead to major changes in productivity.

It is currently fairly difficult to make international comparisons between productivity indicators. If one wants to make comparisons between figures produced in different countries, it should be done using EU KLEMS or OECD data as a basis or by carefully studying productivity calculations in the countries concerned. The methods may differ from each other even if they were based on KLEMS.

In productivity surveys, percentage changes are indicated using a natural logarithm, which means that they may differ slightly from ordinary percentage changes. By using logarithmic rates of change, we can sum up the impacts of different components.

Appendix 1. Concepts and definitions.

Only investments and the average lifetime of assets are considered in the calculation of **gross stock**. Investments are made in specific years. Investments boost the gross stock. An investment remains in the gross stock until the end of the average lifetime of the asset in question. After that it is removed from the stock. This is known as depreciation.

In the **net stock**, consideration is also given to the fact that an asset loses some of its value during use. A certain proportion of the asset is removed from the net stock it comprises each year. This is called consumption. The consumption profile determines how much of the asset value is removed each year. The consumption profile is also known as the age-price profile. This specifically refers to the fact that as the asset ages, it can no longer be sold at the same price as in the past – it is no longer of equal value.

Multi-factor productivity is obtained as a residual when the impacts of changes in labour and capital are subtracted from the change in value added. The calculation of multi-factor productivity is also known as growth accounting, because the growth in output/value added in it is broken down into components.

To measure multi-factor productivity, we must examine how the quality and amount of labour input change. The amount of labour input is measured as the number of hours worked. The quality of labour input is measured with the quality of labour indicator. Capital input is measured with capital services.

Labour productivity is calculated by dividing gross domestic product (value added) or output by the number of hours worked to achieve it. According to the neoclassical economic theory, economic growth is born out of growth in labour productivity.

Multi-factor productivity refers to the part of growth in value added that cannot be explained by the growth in inputs (capital, labour). However, because this cannot be measured directly, its impacts have to be assessed as a residual, when other factors influencing the growth of value added have been eliminated. Usually, multi-factor productivity is thought to be almost the same as technical development, but it may also include other factors. The term is known as multi-factor productivity because it describes productivity-to-total input ratio.

In the methodological description, multi-factor productivity was marked with A. A can be solved with equation 3, resulting in:

$$A = \frac{Y}{K^{v_K}(hL)^{v_L}} \qquad \text{(x)}$$

This is the formula for multi-factor productivity. Thus, A is the productivity with regard to all production inputs (total input) (all production inputs are listed under denominator, while the output is listed under the numerator). This explains the term multi-factor productivity, which often causes confusion.

Multi-factor productivity can be calculated on the basis of value added or output. It can be obtained when the impacts of contributions of capital and labour force are subtracted from the change in value added (or output). If the calculation is based on the output method, the impact of intermediate consumption is also subtracted.

Multi-factor productivity based on value added. By definition, value added is output minus intermediate consumption. When a value added calculation is used to estimate multi-factor productivity, capital and labour contributions (multi-factor productivity) are subtracted from

$$the\ change\ in\ value\ added._{value\ added} = change\ in\ value\ added - (labour + capital\ contributions)$$

Multi-factor productivity based on output. It is justified to assume that there is exchange of intermediate products between industries. For instance, industry x may supply conductors for industry y. The conductors of industry x are the final products of industry x, but not the end products for the whole production chain. Industry y utilises conductors to manufacture its own end products, such as technical devices or other high-tech equipment. When using the value-added method, the contribution of the intermediate input remains unnoticed. The output-based productivity calculation accounts for this effect.

The formula to calculate multi-factor productivity is changed as follows:

$$multi - factor\ productivity_{output} = change\ in\ output - (contributions \text{of capital,}\ labour\ and + intermediate\ \text{consumption} +)$$

Change in the structure of labour input refers to the changes in labour input. In productivity surveys, hours worked and labour compensations are classified according to employees' age, educational level and gender. This allows us to notice what part of productivity growth is caused by changes in these factors.

Employment statistics data starting from 1975 have been used to evaluate changes in the structure of labour input. The data are divided into the above-mentioned categories. A specific weight is determined for each category by utilising average wages and salaries in the category (assuming that wages and salaries describe the marginal productivity of the labour force). For example, one category consists of highly educated persons aged between 30 and 54, who, by their age and education, can be assumed to earn more than a person with upper secondary education aged under 30, for example. If the relative share of highly educated people grows inside the total labour input, or if the relative pay of the highly educated persons rises, this is visible as growth in the structural change term.

The name of the **growth accounting** method comes from annual changes in value added being broken down into growth components, whereby it is possible to examine from which factors growth is derived. It is assumed that from year *t* to year *t–1*, value added has grown by five per cent (logarithmic percentage changes). This growth can be broken down into components: the share of growth caused by the growth in the amount of capital, the share due to growth in labour input and the share resulting from improved multi-factor productivity. The components are summed to the change in value added, that is, if in the example above we assume that the impact of capital is 0.7 per cent and that of labour force 1.3 per cent, the impact of multi-factor productivity is three per cent.

Primary inputs – capital and labour – can be further broken down into sub-items. In productivity calculations, the contributions have been separately calculated for ICT and R&D assets, machinery and equipment, residential buildings and other capital resources. The impacts of hours worked and the contribution of structural change within labour input are separated from labour input.

The most precise productivity survey calculations are made for 63 industries. Value added, labour productivity, contribution of capital and labour force to productivity and multi-factor productivity are calculated for each industry. After this, industry-specific data are aggregated with value added weights to less detailed levels and afterwards to the level of the whole economy.

Contribution of capital intensity. The impact of changes in capital intensity (volume of capital services/hour worked) on the change in labour productivity.

Labour compensation (wages and salaries) is the price of labour input from the perspective of the producer. When examining the impact of the change in the quality of labour force on the change in productivity, the hours worked by employees with different educational levels are weighted by the share of this employee group's compensations of labour force in all compensations of labour force.

When compensations of labour force are subtracted from the industry's value added, capital compensation remains. When examining the impact of capital services on productivity, the amount of capital compensations is used for calculating weights of capital good types by industry (cf. compensations of labour force). The amount of capital compensations is also used in the calculation of the internal rate of return.

Productive capital stock. In the national accounts, the capital stock refers to the amount of capital in the national economy. This includes both physical capital such as machinery, equipment and buildings, and intellectual property products such as research and development.

The productive capital stock used in productivity surveys differs from that used in the national accounts. It measures the capital services provided by the capital stock. The amount of capital services describes the capital stock better than the production capacity of capital and corresponds to other factors of the production function.

Internal rate of return refers to the cost of capital, i.e. internal interest. It is part of the costs of using a capital asset. It can be calculated as a residual when capital compensations, capital gains/losses caused by price changes and the consumption rate of capital are known.

The **depreciation rate** describes how much of the capital stock is used up during the year. Depreciation is the result of normal wear and tear and foreseeable obsolescence, including a provision for losses of fixed assets as a result of accidental damage that can be insured against. In productivity surveys, depreciation rates are specific to industries and asset types. (Consumption of fixed capital should be distinguished from the depreciation shown in business accounts.)

User cost of capital is the price of capital services. It describes the amount of money which would have been needed during the year to cover the use of capital good services to the value of EUR x. This includes capital financing costs or an alternative cost for capital now being unavailable for use elsewhere in production and gains or losses caused by price changes in the capital good and consumption of capital in use.